Topic: Classical mechanics (Moment of Inertia)
Course: B.Sc/ Physics

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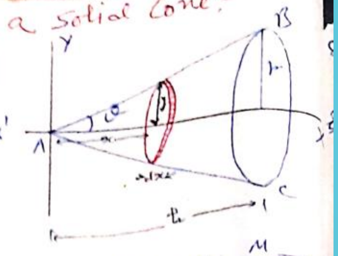
Moment of inertia of a solid cone:-

(i) About an axis passing through its vertex and perpendicular to its axis:-



Let us consider a solid cone ABC of mass M, radius r and height h. $V = \frac{1}{3}\pi r^2 h$ $\therefore \rho = \frac{M}{\frac{1}{3}\pi r^2 h} = \frac{3M}{\pi r^2 h}$.

The cone is rotating about the axis XAX'. Let us consider an elementary disc of the cone. The elementary disc is of thickness dx and is at a distance x from the vertex A of the cone.

$\therefore$ Mass of the element, $dm = \pi y^2 dx \cdot \rho$

Now $\tan\theta = \frac{y}{x} = \frac{r}{h}$ $\therefore y = \frac{xr}{h}$.

$\therefore dm = \pi \frac{x^2 r^2}{h^2} dx \cdot \rho$

Now $dI = \frac{1}{2} dm\, y^2 = \frac{1}{2} \times \pi \frac{x^2 r^2}{h^2} \rho \cdot y^2 dx$

$$= \frac{1}{2}\pi \frac{x^4 r^4}{h^4} \rho\, dx$$

$\therefore$ M.I of the whole cone about the axis XAX'

$$I = \int_0^h dI = \frac{1}{2}\pi \frac{r^4}{h^4}\rho \int_0^h x^4 dx = \frac{1}{2} \cdot \frac{\pi r^4}{h^4} \rho \times \frac{h^5}{5}$$

$$= \frac{1}{10} \pi r^4 h \rho = \frac{1}{10} \cancel{\pi} r^{\cancel{4}\,2} \cancel{h} \times \frac{3M}{\cancel{\pi} \cancel{r^2} \cancel{h}} = \frac{3}{10} M r^2$$

If K is the radius of gyration, then $K = \sqrt{\frac{3}{10}} \cdot r$

(ii) About an axis passing through its vertex and parallel to its base:-

In case (i), M.I of the elementary disc about its diameter $= \frac{1}{4} dm \times y^2 = \frac{(\pi y^2 dx \rho) y^2}{4}$

By parallel axes theorem, M.I of the disc about YAY' $= \frac{(\pi y^2 dx \rho) y^2}{4} + (\pi \rho y^2 dx) \cdot x^2$

$$= \pi \rho \left(\frac{y^4}{4} + y^2 x^2 \right) dx.$$

$\text{Slope} = \frac{x \cdot r}{h} \quad \therefore \; dI = \pi\rho\left(\frac{x^4 r^4}{4h^4} + \frac{x^4 r^3}{h^2}\right)dx$

$$\therefore I = \int_0^h dI = \int_0^h \pi\rho\left(\frac{x^4 r^4}{4h^4} + \frac{x^4 r^2}{h^2}\right)dx$$

$$= \pi\rho\left[\frac{r^4}{4h^4}\left(\frac{h^5}{5}\right) + \frac{r^3}{h^2}\left(\frac{h^5}{5}\right)\right] = \pi\rho\frac{h r^3}{5}\left[\frac{r^2}{4} + h^2\right]$$

$$= \pi \times \frac{3M}{\pi r^2 h} \times \frac{h r^2}{5}\left[\frac{r^2}{4} + h^2\right] = \frac{3M}{5}\left(r^2/4 + h^2\right).$$

$$\therefore I = MK^2 = \frac{3M}{5}\left(\frac{r^2}{4} + h^2\right) \quad \therefore \; K = \sqrt{\frac{3}{5}\left(\frac{r^2}{4} + h^2\right)}$$

Hence, the gradient of a scalar field is a vector field, the vector at any point having a magnitude equal to the maximum rate of ~~increase~~ change $(d\phi/ds)_{max}$ of ϕ at that point and its direction is perpendicular to the surface ϕ = constant.

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