Physics
Theory Part 20

Topics: Classical Mechanics/ Thermal Physics
Course: B.Sc/ Physics

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Classical Mechanics, show that $\frac{dL}{dt} = \sum \frac{d}{dt}(p_j \dot{q}_j)$

Virtual work $\delta W = \vec{F}_i \cdot \delta \vec{r} = 0$ | $\vec{F}_i = \vec{F}_i^a + \vec{f}_i$

$\sum_{i=1}^{N} (\vec{F}_i^a + \vec{f}_i) \cdot \delta \vec{r}_i = 0$ or, if $\delta \vec{r}_i \perp \vec{f}$, then $\vec{f} \cdot \delta \vec{r}_i = 0$.

$\sum_i \vec{F}_i^a \cdot \delta \vec{r}_i = 0$. | $\vec{F}_i = \dot{\vec{p}}_i$ or, $\vec{F}_i - \dot{\vec{p}}_i = 0$.

$\sum_{i=1}^{N} (\vec{F}_i^a - \dot{\vec{p}}_i) \cdot \delta \vec{r}_i = 0$. Total virtual work done by the effective force is zero / Stable equilibrium
D'Alembert's Principle

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$ Non potential force, $\vec{F}_{cor} = 2m(\vec{v} \times \vec{\omega})$

$\vec{F} = q(\vec{v} \times \vec{B})$

For dissipative force $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = 0$ | R = Rayleigh Dissipation formula

$\vec{F} = \cancel{\nabla V} - \Delta V$ | $\frac{\partial L}{\partial \dot{q}_j} = \frac{\partial}{\partial \dot{q}_j}(T - V) = \frac{\partial T}{\partial \dot{q}_j} - \underbrace{\frac{\partial V}{\partial \dot{q}_j}}_{0}$ $V \neq V(\dot{q}_j)$

or, $\frac{\partial L}{\partial \dot{q}_j} = \frac{\partial}{\partial \dot{q}_j} \sum_j \frac{1}{2} m_j \dot{q}_j^2 = m_j \dot{q}_j = p_j \Rightarrow \boxed{p_j = \frac{\partial L}{\partial \dot{q}_j}}$ Generalised momentum

$\frac{d}{dt}(p_j) = \frac{\partial L}{\partial q_j} = 0$. If q_j is cyclic, $\frac{\partial L}{\partial q_j} = 0$.

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$. If $L \neq L(q_j) \Rightarrow \frac{\partial L}{\partial q_j} = 0$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0 \Rightarrow \frac{\partial L}{\partial \dot{q}_j} = p_j = \text{Const.}$ | $L = L(q_j, \dot{q}_j, t)$

$\frac{dL}{dt} = \sum_j \frac{\partial L}{\partial q_j} \frac{dq_j}{dt} + \sum_j \frac{\partial L}{\partial \dot{q}_j} \frac{d\dot{q}_j}{dt} = \sum \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) \frac{dq_j}{dt} + \sum_j \frac{\partial L}{\partial \dot{q}_j} \frac{d\dot{q}_j}{dt}$

$= \sum_j \frac{d}{dt}\left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = \sum_j \frac{d}{dt}\left(\dot{q}_j \frac{\partial T}{\partial \dot{q}_j}\right) = \sum_j \frac{d}{dt}(\dot{q}_j p_j)$

If the energy of two Black bodies is $E_1 = 3.82 E_2$, find relation between T_1 & T_2.

By Stefan-Boltzmann law for black body is

$$E \propto T^4$$

$$= \sigma T^4; \quad \sigma = \text{S.B Const}$$

$$\therefore \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 \qquad \cancel{T_2 = 4}$$

$$E_1 = 3.82 E_2$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{1}{3.82}\right)^{1/4} = \frac{1}{0.714}$$

$$= 1.4 \Rightarrow \boxed{T_2 = 1.4\, T_1}$$

So, In Black Body, Energy depends only on the temperature (T) of the body.

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