Physics
Theory Part 16

Topics: Properties of Materials/ Differential Equations

Course: B.Sc/ Physics

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Write the Doomsday differential equation & find its solution for condition at $t=0$, $y(t)=y_0$. Why it is called Doomsday equation.

Doomsday equation & its solutions:

$$\frac{dy}{dt} = y^{1.01}$$ by separation of variable Method of integration

$$\int y^{-1.01}\,dy = \int dt + c$$ (where c is a constant determined by boundary conditions).

or, $y^{-0.01} = -\frac{(t+c)}{100}$ — ① let at $t=0$, $y(t)=y_0$

After substitution of this in eqn ①

$$y_0^{-0.01} = -\frac{c}{100} \Rightarrow \boxed{c = -\frac{100}{y_0^{0.01}}}$$ putting this in eqn ①

$$y^{-0.01} = -\frac{\left[t - \frac{100}{y_0^{0.01}}\right]}{100}$$ or, $y^{1/100} = \frac{-100}{\left[t - \frac{100}{y_0^{0.01}}\right]}$

or, $\boxed{y(t) = \left[\frac{-100}{t - \frac{100}{y_0^{0.01}}}\right]^{100}}$ Final solution of the differential eqn. — ②

from eqn. ②, it is clear that when $t = \frac{100}{y_0^{0.01}}$ then $y(t) \to \infty$ i,e it diverges.

If we knew the value of constant y_0, we can calcute time, t in which the population will increase to its infinite value i,e explodes.

This eqn. is called Doomsday is because the solution $y(t)$ blows upto infinity upon reaching a certain threshold value ~~to~~ of t.

Also in eqn. $y' = y^{1.01} = y^{1+0.01} = y^{1+\varepsilon}$ ($\varepsilon > 0$) so power of y is greater than one, so growth rate is more than natural one i,e $y' = y$. It is abnormal growth.

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Difference between Stress & Strain.

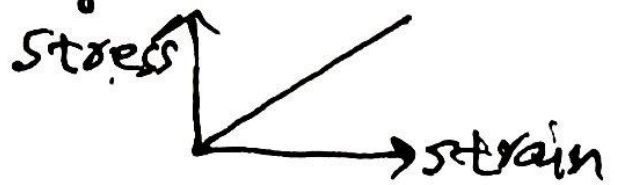
Stress: It comes from the word 'destresse' in English, which means 'distress'. Quantitatively, it is the force applied per unit area of the body and produces changes in its shape, size, vol. or length of the body. $\text{stress}(\sigma) = \frac{\text{force}(F)}{\text{Area}(A)}$ (N/m^2).

Strain: It is the effect of the stress (cause) applied on the body. It is a dimensionless quantity, it is measured as a ratio of change in dimension (shape) per original shape.

$$\text{strain}(\epsilon) = \frac{\Delta L}{L_0} \quad \text{where } \Delta L = L - L_0$$

within elastic limit, $\text{stress}/\text{strain} = \text{Const.}$ (Y - Young's Modulus for length)

strain is fundamental so plotted along x-axis.



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