Physics
Theory Part 14

Topics: Quantum Physics/ General Properties of Matter

Course: B.Sc/ Physics

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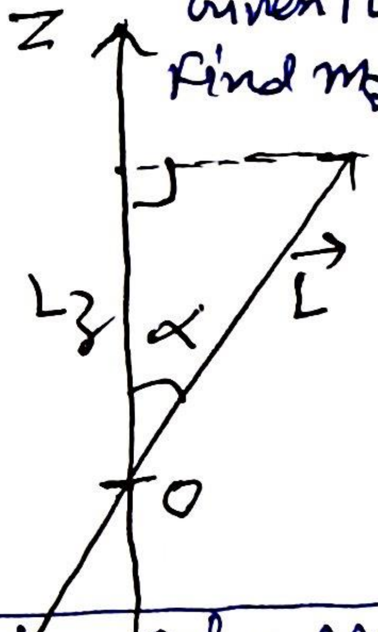
Given $|\vec{L}| = \sqrt{56}\hbar$ Find m_l & α



Atomic vector model for $|\vec{L}| = \sqrt{56}\hbar$

Now $|\vec{L}| = \sqrt{l(l+1)}\,\hbar$ & $L_z = m_l \hbar$ from vector model of an atom and

$$\cos\alpha = \frac{L_z}{|\vec{L}|} = \frac{m_l}{\sqrt{l(l+1)}} \therefore \alpha = \cos^{-1}\left(\frac{m_l}{\sqrt{l(l+1)}}\right).$$

Given $L = |\vec{L}| = \sqrt{56}\,\hbar = \sqrt{l(l+1)}\,\hbar \Rightarrow l = 7$

corresponding m_l values are given by

$-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7$

Minimum value of $m_l = -7 \Rightarrow \alpha = \cos^{-1}\left(\frac{-7}{\sqrt{56}}\right) = \cos^{-1}(-0.935) = 159.2^\circ$

Maximum " " $m_l = +7 \Rightarrow \alpha = \cos^{-1}\left(\frac{7}{\sqrt{56}}\right) = \cos^{-1}(0.935) = 28.7^\circ$

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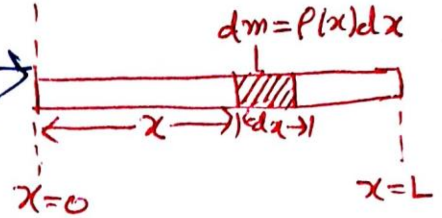
Thus $\alpha_{min} = 20.7^\circ$ & $\alpha_{max} = 159.2^\circ$.

From Definition of Centre of mass is — find x_{cm} for $\rho(x) = \rho_0 + \frac{(\rho_1-\rho_0)x^2}{L^2}$ →



$$x_{cm} = \frac{\int_{x=0}^{L} x\,dm}{\int_{x=0}^{L} dm}$$

Given $\rho(x) = \rho_0 + \frac{(\rho_1-\rho_0)}{L^2}x^2$

or, $\rho(x) = A + Bx^2$

where $A = \rho_0$, $B = \left(\frac{\rho_1-\rho_0}{L^2}\right)$

$$= \int_{x=0}^{L} x\rho(x)dx \Big/ \int_{x=0}^{L} \rho(x)dx$$

$$= \int_{x=0}^{L} x(A+Bx^2)dx \Big/ \int_{x=0}^{L} (A+Bx^2)dx$$

$$= \int_{x=0}^{L} (Ax+Bx^3)dx \Big/ \int_{x=0}^{L} (A+Bx^2)dx$$

$$= \left[\frac{A}{2}x^2 + \frac{B}{4}x^4\right]_0^L \Big/ \left[Ax + \frac{B}{3}x^3\right]_0^L$$

$$= \left(\frac{A}{2}L^2 + \frac{B}{4}L^4\right) \Big/ \left(AL + \frac{B}{3}L^3\right) = \frac{L^2\left(\frac{A}{2} + \frac{B}{4}L^2\right)}{L\left(A + \frac{B}{3}L^2\right)} = \frac{\frac{L}{2}\left(A + \frac{B}{2}L^2\right)}{\left(A + \frac{B}{3}L^2\right)}$$

$$= \frac{\frac{L}{2}\left[\rho_0 + \frac{\rho_1-\rho_0}{2}\right]}{\left[\rho_0 + \frac{\rho_1-\rho_0}{3}\right]} = \frac{\frac{L}{2}\left(\frac{\rho_1+\rho_0}{2}\right)}{\left(\frac{2\rho_0+\rho_1}{3}\right)} = \boxed{\frac{3L}{4}\left(\frac{\rho_1+\rho_0}{\rho_1+2\rho_0}\right)}$$

FOR ANY QUERIES FEEL FREE TO CONTACT ME AT EMAIL: RAJESH.NEOGY@GMAIL.COM

Thanksss